STABILITY ANALYSIS OF CONCRETE GRAVITY DAM USING FEM

Dragan RAKIĆ[1]*, Milan BOJOVIĆ[1], Snežana VULOVIĆ[1], Miroslav ŽIVKOVIĆ[1], Dejan DIVAC[2], Nikola MILIVOJEVIĆ[2]

[1]Faculty of Engineering, University of Kragujevac,
Sestre Janjić 6, 34000 Kragujevac, Serbia; drakic@kg.ac.rs

[2]Jaroslav Černi Institute for the Development of Water Resources,
Jaroslava Černog 80, 11226 Pinosava-Belgrade, Serbia; ddivac@eunet.rs

Abstract

Analyses of mechanical, thermal and filtration processes in the large gravity dam and surrounding rock mass were performed and presented. The analyses were conducted within the stability analysis of the dam Djerdap 1. The dam was thoroughly modelled according to the project documentation in order to detect potential flaws or failure of specific parts of dam system. Additionally, it is possible to analyze dam stability during potential failure of specific parts of drainage system. Since the dam lies on a very complex fundament, material characteristics are assigned to each finite element according to geological maps obtained through testing of materials. All the loads influencing the dam in exploitation conditions (mechanical, thermal and filtration loads) are applied to the model. In order to consider the interaction of different processes, a coupled analysis was conducted. After the preliminary FEM analyses, obtained results were compared to the measured values, upon which the input parameters where accordingly corrected. For such adopted input parameters, dam safety factors were calculated in repeated analyses. Model calibration was not performed.

Keywords: Dam safety; Stability analysis; Gravity dam, Constitutive modelling; FEM

1 Introduction

Concrete gravity dams are among the most important infrastructure facilities whose potential damages could cause catastrophic economic consequences and loss of human lives. Periodic analysis of the dam stability is a very important issue for its safe operation during long period of exploitation.

Dam Djerdap 1, located on the Iron Gate gorge between Serbia and Romania, is the largest dam on the river Danube and one of the largest hydro power plants in Europe. Hydroelectric power station Djerdap 1 was built on 943 km from the mouth of the Danube to the Black Sea, and it forms a reservoir volume of 1,380 million m³. The dam is symmetrical, with overflow section and locks in the middle of the Romanian and Serbian side (Figure 1).

Main goal of presented analysis was to create a numerical model, conduct preliminary analysis and develop a procedure for the existing state of the object according to tridimensional geometry. In order to determine the state of the object after more than 45 years of exploitation, it is necessary to consider all the loads influencing the dam and to conduct the following analyses: analysis of thermal processes in concrete, analysis of filtration processes as well as stress-strain analysis. All three types of loads influence simultaneously so it is necessary to conduct coupled analyses. During FE model preparation, all constructive elements of overflow dam (dilatation joint, sealing elements, drainage system, anchors) were taken into account in order to realistically analyze all the physical processes occurring in the dam and surrounding rock mass. Grouping of surrounding rock mass in quasi-homogenous zones was conducted for defined materials. Appropriate elasto-plastic models for rock mass were used for mechanical behavior modelling.

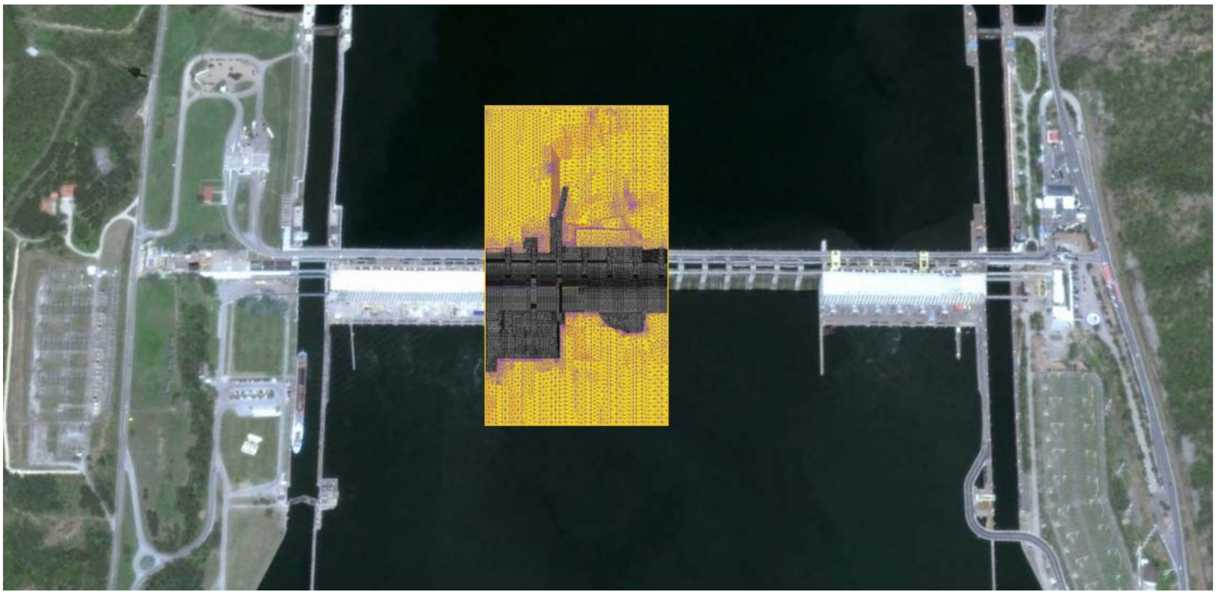

Figure 1. Dam Djerdap 1 and overflow section FE model

Within the presented analysis, preliminary calculations were conducted using material parameters determined during the object construction. In other words, neither the change of material characteristics during the time was considered nor the model calibration was performed.

2 Theoretical Background

This section presents theoretical bases of solving the problem of heat conduction, fluid flow through porous media and strength analysis of solids [1]. Presented theoretical bases were implemented into PAK program [2] and used during the analysis of dam stability.

2.1 Governing equations of heat transfer

Governing equation of heat equilibrium is based on the fundamental principle of heat conservation [1, 3]. Change of material internal energy in the time unit, in elementary volume dV, is equal to the amount of thermal energy accumulated in the same volume in the unit time, or

$$\frac{dQ}{dt}=\frac{dU}{dt} \tag{1}$$

where dQ and dU are the changes of thermal and internal energy in the volume dV in elementary time interval dt. Change of internal energy can be formulated as

$$\frac{dU}{dt}=\rho c\frac{dT}{dt}dV \tag{2}$$

where: ρ - material density, c - specific heat, T - temperature, t - time.

According to the Fourier's law, used in the analysis of heat transfer, a relation between heat flux $\mathbf{q}$ and temperature gradient is established ∇T and formulated as

$$\mathbf{q}=-\mathbf{k}\nabla T \tag{3}$$

where $\mathbf{k}$ represents a transfer or conductivity matrix. Using equations (1) and (3), a governing equation of heat transfer is obtained as

$$-\rho c\frac{dT}{dt}+\nabla^{T}\left(\mathbf{k}\nabla T\right)+q=0 \tag{4}$$

General solution of governing equation of heat transfer contains undetermined functions and constants. Practical problem solving requires a solution for initial and boundary conditions. They indicate that the temperature distribution at the initial time $t=0$ is known. Boundary conditions in general case can be: specified temperature, flux, heat convection or specified emission (radiation).

2.2 Governing equations of filtration

Water in porous media moves through a complex system of pores, specifically through active part only of mutually connected pores. Moving of underground water is practically impossible to consider as a flow through the system of mutually connected pores so a continuum model is introduced for basic equation defining [4, 5].

Basic equation, which defines the law of underground water flow, was defined by Darcy. Darcy's law can be formulated in the equation

$$\mathbf{q}=-\mathbf{k}\mathbf{i} \tag{5}$$

where $\mathbf{k}$ is a permeability matrix and $\mathbf{i}$ is a potential gradient φ

$$\mathbf{i}=\nabla\varphi \tag{6}$$

Hydraulic potential is defined as

$$\varphi = \frac{p}{\gamma} + h \tag{7}$$

where p is the pore pressure of the fluid, γ is the unit weight of fluid, whereas h is the height of measured point in relation to referent plane.

According to the continuity equation and Darcy's law (5), a hydrodynamic equation for steady state flow is formulated as

$$\nabla^T (\mathbf{k} \nabla \varphi) + Q = S \frac{\partial \varphi}{\partial t} \tag{8}$$

where Q is the volumetric flux, S is the effective layer porosity or specific bounty for limited flow. Boundary conditions in solving the flow problem through porous media as previously mentioned though equations could be: specified potential and/or specified flux.

2.3 Governing equations of strength analysis

Analysis of stress-strain processes determines the displacement, strain, stress and other relevant variables introduced as internal variable [1, 3]. Starting value in the continuum mechanics is the displacement $\mathbf{u}$.

In static conditions, the following equilibrium condition must be fulfilled in each material point

$$\nabla^T \boldsymbol{\sigma} + \mathbf{F}^V = 0 \tag{9}$$

where $\boldsymbol{\sigma}$ is the strain tensor, whereas $\mathbf{F}^V$ is the vector of body forces [1]. Contour conditions can be: specified displacements and/or specified surface forces.

Using equilibrium equation (9) and contour conditions, a virtual work principle is derived representing a basic equilibrium equation in mechanic of deformable body

$$\int_V \boldsymbol{\sigma} \delta \boldsymbol{\varepsilon} dV = \int_V \mathbf{F}^v \delta \mathbf{u} dV + \int_{S^\sigma} \mathbf{F}^s \delta \mathbf{u} dS \tag{10}$$

Apart from the presented equilibrium equations, it is also necessary to define constitutive relations for solving problems in soil mechanics. In the analysis of the dam stability, Hoek Brown material model was used for simulation of mechanical behavior of the surrounding rock mass [6, 7]. Yield surface of this model is the function of stress state and can be formulated using stress invariants as:

$$f = \frac{I_1}{3} m_b \sigma_{ci}^{\left(\frac{1}{a}-1\right)} - s\sigma_{ci}^{\frac{1}{a}} + 2^{\frac{1}{a}} \left(\sqrt{J_{2D}} \cos\theta\right)^{\frac{1}{a}} + m_b \sqrt{J_{2D}} \sigma_{ci}^{\left(\frac{1}{a}-1\right)} \left(\cos\theta - \frac{1}{\sqrt{3}} \sin\theta\right) \tag{11}$$

Values I_1 and J_{2D} in the equation (11) represent stress invariants while the value θ represents Lode's angle. Values σ_{ci}, m_b, s and a are the parameters of material model.

For describing of mechanical behavior of the dam, material model of concrete defined by one parameter was used [8].

3 Safety Evaluation Method

3.1 Safety factor evaluation

Global safety factor of the object represents a ratio of shear strength and shear stresses in a material [9, 10]. In other words, dam instability occurs when the shear stress τ in the material exceeds the value of shear strength of the material τ_f, or

$$F = \frac{\tau_f}{\tau} \tag{12}$$

Global safety factor represents a maximal value of shear strength reduction factor securing the object stability or indicating the convergence of numerical solutions.

4 FE Analysis of Concrete Gravity Dam

Analyzed dam is equipped with instruments representing the system of technical observation and monitoring changes in the dam. Monitored values are displacements, slope changes, temperature, pore pressure, precipitation, water level, underground water level, pump performance, dilatations and crack performance, relative displacement of construction parts, forces in the framework, concrete deformations, etc. Basic constructive dam elements and their modelling are discussed below.

4.1 Construction details

Dilatations represent a gap between adjacent sections of the dam and between dam and waterfall. They provide relative displacement of adjacent dam sections in all three directions. In order to provide these conditions in FE model, nodes between the sections are independent.

In order to analyze leakage through waterstops, conductible elements were generated in the planes (2D elements) of dilatations. In order to simulate water penetration through certain parts of waterstops, waterstops are organized as shown on Figure 2. Conductible 1D elements were created in the gallery and manhole area to simulate the flow of leachate. These elements were also used for drainage wells under the dam body as well as canals for water evacuation.

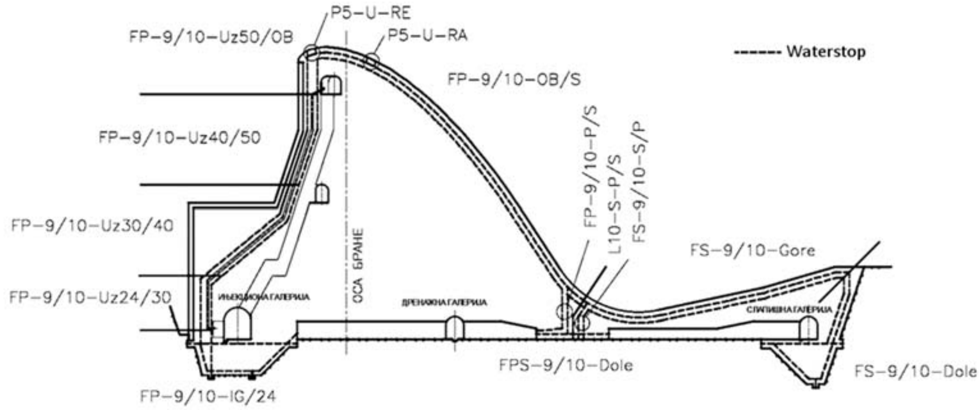

Figure 2. Waterstops between two sections

Apart from filtration calculations, in strength calculation, 1D finite elements were used for steel anchor modelling that connecting dam waterfall with rock mass.

4.2 FE model of the dam

FE model was formed according to tridimensional geometrical model of overflow dam, geological material map and exploitation conditions. Tetragonal 3D finite elements with midside nodes were used for modelling concrete dam and surrounding rock mass, whereas 2D finite elements used for modelling of dilatation joints. As previously stated, 1D finite elements were used for drainage system and steel anchors modelling. FE model consists of 1472730 nodes and 1063038 elements (Figure 3).

In the dam body, elements of 2m average size were generated whereas the element size increases from 5 to 10m in the rock mass. This provides larger elements on model boundaries far from the dam construction.

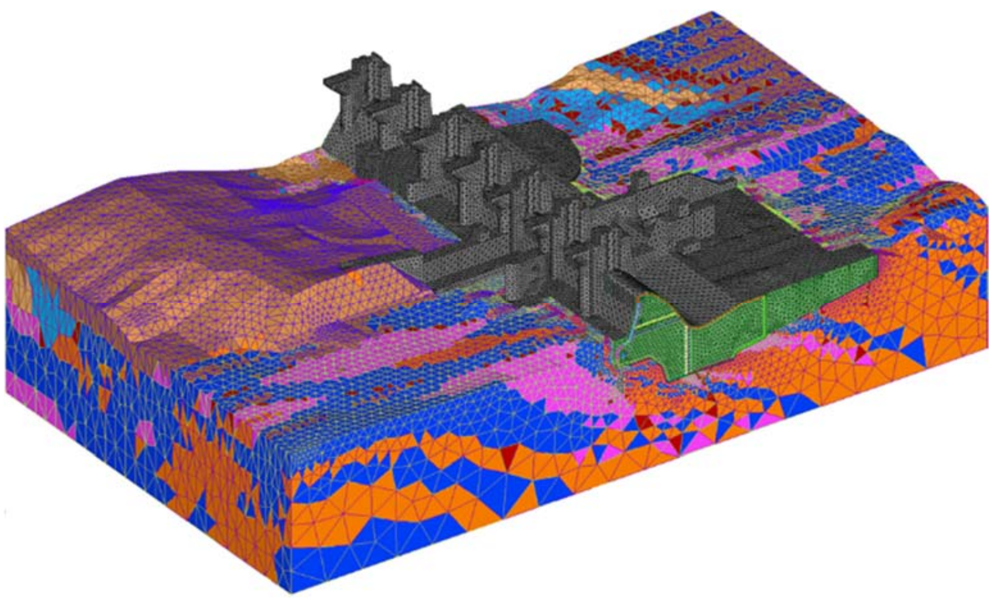

Figure 3. FE model of the overflow section of the dam

In first pass rock mass is generated as one quasi-homogeneous zone. Using external code and geological maps, materials were assigned to existing elements of rock mass elements as shown on Figure 3.

4.3 Loads and boundary conditions

Model loading are defined for exploitation conditions. Thermal, filtration and stress-strain processes were analyzed for following contour conditions: winter temperatures, nominal upper water level, minimal lower water level, drainage system is in function, all waterstops are in function. Natural boundary conditions are applied to the model, whereas the lower model boundary is completely fixed.

4.4 Material parameters

In the analysis of thermal processes in concrete, heat conduction coefficient 10^{-5} 1/K was used. Temperature of wetted dam surface is the function of water depth (Table 1) whereas the concrete temperature in contact with rock mass and with air are constant.

Table 1 Temperature as a function of the water depth

MASL	69,6	61	54	47	40	30	20
Temperature [°C]	2	3.5	5	7.5	11	15	17

Filtration coefficients presented in Table 2 were used for modelling of filtration process in rock mass. Filtration coefficient used for concrete was 10^{-11}m/s.

Table 2 Filtration coefficients of rock mass

Paramete r	**mat 1**	**mat 2**	**mat 3**	**mat 4**	**mat 5**	**mat 6**	**mat 7**
k [m/s]	$6x10^{-8}$	$9x10^{-7}$	$2,5x10^{-9}$	$5x10^{-8}$	$4x10^{-8}$	$1,8x10^{-8}$	$5x10^{-4}$

For modelling of rock mass mechanical behavior Hoek-Brown material model was used. Parameters of material model is shown in Table 3.

Table 3 Material parameters of rock mass

Paramete r	mat 1	mat 2	mat 3	mat 4	mat 5	mat 6	mat 7
RMR	67	46	37	31	21	16	16
m_i	28	20	15	12	12	12	12
σ_{ci} [MPa]	83	75	58	48	40	20	20
E [MPa]	6000	4000	4000	3000	1500	600	600
m_b	8,616	2,907	1,581	1,021	0,714	0,597	0,597
s	0,0256	0,0025	0,0009	0,0005	0,0002	0,0001	0,0001
a	0,502	0,508	0,514	0,521	0,541	0,557	0,557

All the rock mass material parameters were determined during the dam construction and are not calibrated at this stage of the project.

4.5 Analysis procedure

The analyses of thermal, filtration and stress-strain processes in the dam and surrounding rock mass were conducted. For defined contour conditions, thermal and filtration calculation were conducted first. Results of this calculations were used as contour conditions for stress-strain analysis. During strength analysis, initial stress state was first generated considering that accumulation was empty and obtained strains and displacement were reset. After that, strain analyses for defined load cases were conducted. All the analyses were conducted for steady state conditions.

5 Analysis of Results

This chapter presents the results of thermal, filtration and stress-strain analysis for defined calculation situation. Results are presented in the dam cross-sections where measuring instruments are located and are to be used for model calibration.

5.1 Thermal analysis results

Analysis results of thermal processes for steady-state conditions in two specific dam cross-sections are presented on Figure 4. Temperature field of concrete used as a load in stress-strain analysis was obtained for defined contour conditions.

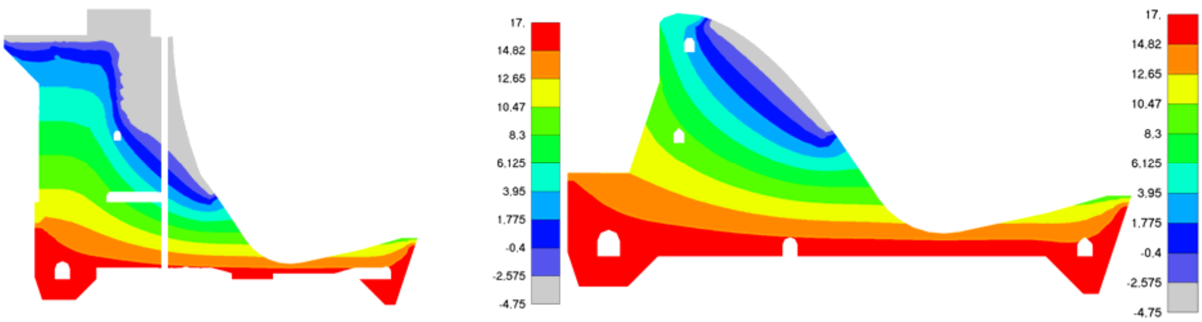

Figure 4. Temperature field in section 2 and 3 [°C]

5.2 Filtration analysis results

Results of filtration analysis for steady-state flow conditions through the dam and rock mass are presented on Figure 5 and Figure 6. According to the conducted filtration analysis, fields of potential, pore pressure, flow velocity and filtration forces were obtained.

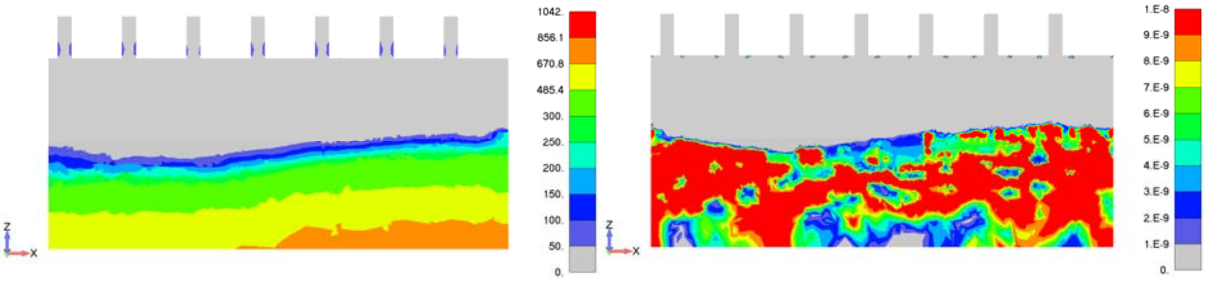

Figure 5. Dam axis section: a) pore pressure [kPa], b) velocity of filtration [m/s]

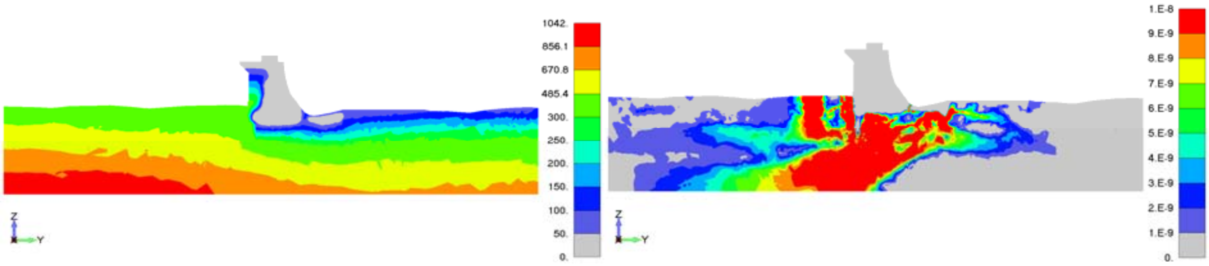

Figure 6. Section 2: a) pore pressure [kPa], b) velocity of filtration [m/s]

Filtration forces obtained through filtration analysis, through the dam and surrounding rock mass, represent loads in the stress-strain analyses.

5.3 Stress-strain analysis results

Figure 7 and Figure 8 present the results of stress-strain analysis of dam section 2. Fields of radial displacement, vertical displacement, total displacement, radial stress, vertical stress are presented.

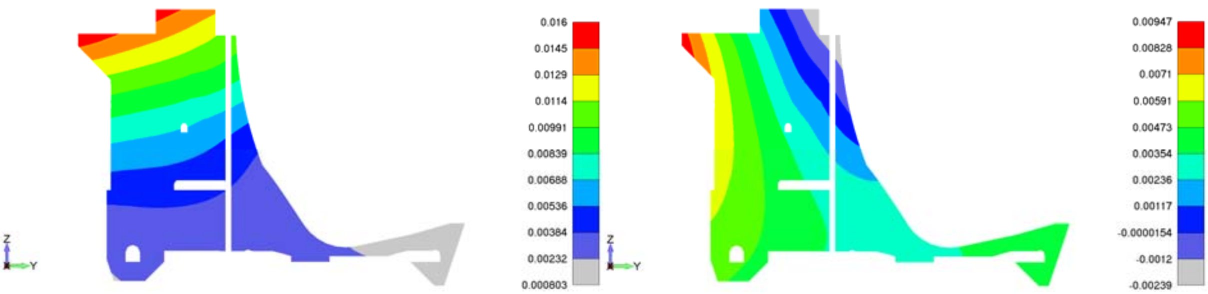

Figure 7. Displacement of section 2: a) radial displacement [m], b) vertical displacement [m]

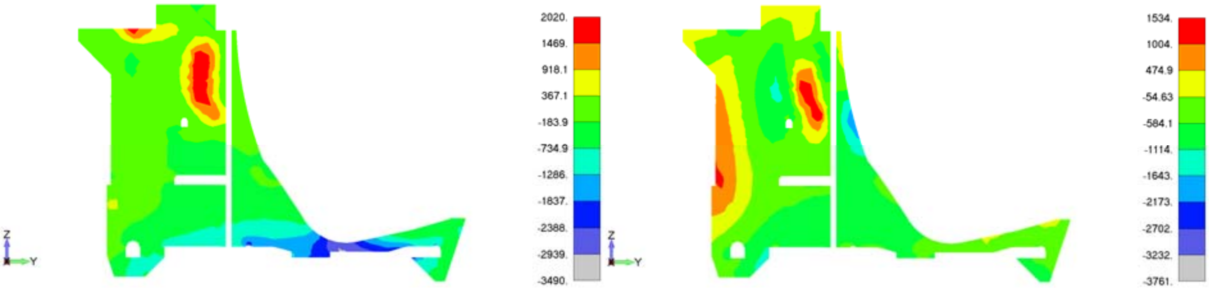

Figure 8. Stress in section 2: a) radial stress [kPa], b) vertical stress [kPa]

Maximal value of total displacement was obtained in the pole section crown and is 22.5mm

5.4 Dam safety factor

After conducting all the analyses, safety factor of construction was determined. In this analysis, thermal and filtration loads were included. Material shear strength was reduced until it was impossible to reach convergence of numerical solutions or until reached the ultimate stability of the dam. Calculated value of global safety factor for the presented case is 5.5. It should be noted that the calculated value of safety factor was obtained for noncalibrated model so this value should be taken with caution.

6 Conclusion

Main goal of the project was to create the finite element model and to conduct analyses of stability in order to determine the current state of the object. In order to determine the current state of the object, after more than 45 years of exploitation, it is necessary to take into account all the loads influencing the dam including: temperature load, filtration load and mechanical load. Accordingly, we conducted the analysis of thermal processes in concrete, the analysis of filtration processes in concrete and surrounding rock mass as well as the strength analysis of concrete dam and surrounding rock mass. All three analyses are coupled so stress-stain analysis apart from mechanical loads influencing the object, takes into account temperature and filtration loads. During the model development, we considered real modelling of all constructing elements of overflow dam (dilatation, sealing elements, drainage system, anchors) in order to realistically analyze all the physical processes occurring in the dam and surrounding rock mass. In order to obtain a realistic distribution of rock mass, geotechnical surroundings were organized in quasi-homogenous zones according to their similar mechanical and filtration characteristics. After generating the finite elements mesh of rock mass, we conducted the material characteristic assignment to the elements of the current mesh using the external code. During the creation of the numerical model, we considered a possible extension in terms of connecting existing model to other structural elements such as power plant and ship locks, whose modelling and analysis are planned. Consequently, model integration into the software system for safety evaluation of the whole dam is enabled. For modelling of the surrounding rock mass and concrete gravity dam tetragonal 3D finite elements with internodes was used, for modelling of dilatations and sealing 2D finite elements was used whereas for drainage system elements and anchors we used 1D finite elements. Within the presented analysis, we conducted preliminary calculations using material parameters determined during the object construction without considering the change of material characteristics during the time. It is necessary to conduct the procedure of model calibration using permanently measured values on the object so that the model behavior would correspond to the object behavior. Model calibration is planned for the future implementation of this project.

Acknowledgements

The authors gratefully acknowledge partial support by Ministry of Science and Technology of the Republic of Serbia, grants TR37013 and TR32036.

References

[1] Kojić M, Slavković R, Živković M and Grujović N, *Finite element method 1, Linear analysis (in Serbian)*, University of Kragujevac, Faculty of mechanical engineering, Kragujevac, Serbia, 1998.

[2] Kojić M, Slavković R, Živković M and Grujović N, PAK-S Program for structure analysis using FEM, University of Kragujevac, Faculty of mechanical engineering, Kragujevac, Serbia, 2016.

[3] Bathe K J, *Finite Element Procedures*, Massachusetts Institute of Technology, USA, 1996.

[4] Schrefler B and Zhan X, A fully coupled model for water flow and airflow in deformable porous media, *Water Resources Research*, *vol. 29*, *1993*, *no. 1*, pp. 155-167, 10.1029/92WR01737.

[5] Lewis R W and Schrefler B A, *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media*, John Wiley & Sons Ltd, West Sussex P019 IUD, England, 1998.

[6] Hoek E, Strength of rock and rock masses, *ISRM News Journal*, vol. 2, 1994, no. 2, pp. 4-16.

[7] Hoek E, Carranza-Torres C and Corkum B, Hoek-Brown failure criterion - 2002 edition, *Proceedings*, Proceedings of the 5th North American Rock Mechanics Symposium and the 17th Tunnelling Association of Canada - NARMS-TAC 2002, Toronto, Canada, 2002, pp. 267-273.

[8] Kotsovos D and Pavlović M, *Structural Concrete: Finite element Analysis for Limit-state Design*, Thomas Telford Publications, London, UK, 1995.

[9] Nian T, Jiang J, Wan S and Luan M, Strength Reduction FE Analysis of the Stability of Bank Slopes Subjected to Transient Unsaturated Seepage, *Electronic Journal of Geotechnical Engineering*, vol. 16, 2011, pp. 165-177.

[10] Rakić D, Živković M, Vulović S, Divac D, Slavković R and Milivojević N, Embankment dam stability analysis using FEM, *Proceedings*, 3rd South-East European Conference on Computational Mechanics ECCOMAS and IACM Special Interest Conference, Kos Island, Greece, 2013.